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VARIABILITY SIMULATIONS OF JOIST FLOOR SYSTEMS

by

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M. E. Criswell and J. Bodig

Structural Research Report No. 13
Civil Engineering Department
Colorado State University
Fort Collins, Colorado 80523

September, 1974

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ABSTRACT

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This thesis examines the use of the Monte Carlo method for studying the effects of component variability on the deflection behavior of wood joist floor systems. The study considers random variations in joist modulus of elasticity within a lumber grade and evaluates this effect on floor deflection behavior. Simulation results indicate that there are two basic effects induced on deflection behavior by joist modulus of elasticity variability. These effects are changes in mean maximum floor deflection and maximum floor deflection variability. A means for seeking optimum economic efficiency through restricting component variability to a value that yields the best floor maximum deflection response to component cost relationship can be formulated from floor maximum deflection distributions. The study emphasizes that important roles of structural interaction and component variability on structural performance. For floors in which a deflection criterion governs design, the design calculation is normally based on the deflection behavior of joists with average member stiffness acting alone. This method of design normally does not include the beneficial effects of load sharing and composite action nor the detrimental effects of component variability. Design analysis based upon a joist-acting-alone behavior assumption does not necessarily describe the behavior of floors within the design. The effects of structural interaction and component variability need to be evaluated for floor behavior to be accurately predicted.

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CHAPTER I

INTRODUCTION AND LITERATURE REVIEW

1.1 Introduction

Efficient utilization of natural resources should constitute a high priority in the design and construction of engineering structures for both ecological and economical reasons. Efficient design of structures is based upon the selection of component dimensions, materials, and connections that are best suited for a particular application. Thorough understanding of system performance is essential for determination of the optimum combination of interacting parameters. Response of a structure as an integrated unit to service conditions must be evaluated with respect to the contribution of individual components. Load-sharing, composite action, and other forms of structural component interdependence alter system response from the performance predicted for individual components acting separately. To efficiently employ resources in the fabrication of a structure, prediction of system behavior which includes the forms of structural interaction must be possible.

Further, natural variation in the dimensions and properties of system components modify the behavior of a population of systems from that of a single system considered on a design basis. The margin of conservatism incorporated into a design is dependent upon the effect on system response caused by component dimension and property variations within the tolerances permitted in the manufacturing process and/or the material property values.

Efficiency of a structure with respect to variability of the components should be considered in conjunction with the effects of system structural interaction. Increasing or decreasing the variability of

components within a system yields corresponding change in the variability of system performance. A tradeoff exists between the cost of restricting the magnitude of component variability and the benefits accruing from the lower variability. The benefits to be gained are (1), lower variability of the system, allowing lower conservatism in design while producing higher system reliability, and, (2), a higher level of structural performance.

Wood joist floor systems, such as those used in residential housings, are complex structural systems. Interaction of joists, sheathing, and connectors yields load-deflection behavior that differs significantly from that predicted for a joist only structure. Variation in the properties or dimensions of materials, connectors or loading results in a distribution of floor response. Changes in the magnitude of variability of any of the floor or loading parameters alter the resulting distributions of floor behavior.

In wood joist floors, the joists and sheathing interact. This interaction is by composite action of the joist and sheathing and by load sharing between the joists resulting from the distribution of load by the sheathing. Current design practice (1) does not include the benefits of composite action and load sharing, except for a 15% increase in allowable stress for repetitive members. Rather, for deflection it is based upon a maximum limit for a single joist. The joist is considered to have a modulus of elasticity corresponding to the mean modulus of elasticity of the lumber grade and is loaded corresponding to a uniform live load of 40 psf. A design must also satisfy a stress criterion, for which some compensation is included for load sharing and composite action of repetitive members. In any given real floor the modulus of elasticity of the

joists will vary according to the characteristics of the lumber population and, if interaction with other components of the floor system is not considered, some joists would be expected to deflect in excess of the joist-only deflection prediction. The deflection characteristics of a population of floors with respect to design practices will depend on the relative magnitudes of the counteracting effects of structural interaction and component variability.

1.2 Objective

This study is part of a research project being conducted at Colorado State University sponsored by the National Science Foundation. Other parts of the project have been directed toward formulating and verifying mathematical models of layered beams with interlayer slip and residential wood joist floor systems.

The objective of this study is to evaluate the potential of using Monte Carlo analysis to investigate the effects of component or loading variations on the performance of wood joist floor systems. The method analyzes the effects of variability and changes in variability on structural behavior using a mathematical model that incorporates system interaction. The potential for higher economic efficiency is examined. The means of increasing efficiency could manifest itself in terms of modifying the variation of properties within lumber grades or changing existing design practice to more efficiently utilize the existing grades.

1.3 Literature Review

Investigation of the effects of changing input parameter variability on structure performance has been very limited to date, particularly as applied to wood joist systems. Several authors have indicated the potential usefulness of a study of this type.

Galligan and Snodgrass (2) discussed the potential of increased lumber efficiency through the use of both more accurately segregated lumber and design practices intended for that lumber. They indicated the potential of use-oriented joist grades for the housing industry and the current uses of machine stress rated lumber for trusses and laminated beams. They also indicated that, to compliment methods of more accurately predicting behavior of wood structures, better definitions of material property values are needed, including the reduction of the variability within a grade.

Polensek (3) developed a model of the static and dynamic deflections behavior of wood joist flooring systems using a finite difference solution technique. He recommended as a possible application of the model a deflection behavior study of floors consisting of randomly selected joists of a given probability distribution. He indicated the resulting population distribution of floor maximum deflections could be compared to the deflection of the same floor as specified by the simplified method of floor design.

Bonnicksen and Suddarth (4) have investigated the effects of variability upon structural reliability of load-sharing systems. In their research, they compared the reliability of single joist members to vertically laminated three joist members. From reliability functions of each group and load probability distributions, a set of structural reliabilities were obtained. Averaging effects of the modulus of rupture of three joists resulted in a lower coefficient of variation of strength and a higher structural reliability. However, they found the mean strengths of three joist members lower than that of single joist members. From this work relative load carrying capacity of the members was evaluated.

Zahn (5) addressed the problem of variations in the strengths of joist structures to study the benefits accrued from load sharing, specifically the effects of grouping several joists together and mutually constraining the joists. The structure studied consisted of N number of joists with a single deck element across the center of the joists. Load sharing, but no composite action, existed in the structure. Zahn considered three models for the joist structures. These were brittlest link, weakest link, and flexible decking models. In the brittlest link structures all joists were required to have the same deflection, a condition equivalent to complete load sharing, whereas, in weakest link structures all joists carried the same load, and consequently no load sharing occurred. The flexible deck model represented the more realistic case between no load sharing and complete load sharing.

From modulus of rupture data for joists of four lumber grades, Zahn (5) derived modulus of rupture distributions for the three models. The brittlest link and weakest link model distributions were obtained analytically. The flexible decking model distribution was generated through a computer simulation.

From a comparison of the distributions for one and five joist structures, Zahn (5) concluded that the maximum possible minimum load capacity increase as a result of load sharing is 12%. This is based on the increase of the brittlest link floors strength over that of the weakest link floors. The flexible deck floors are between the weakest link and the brittlest link floors, and they represent partial load sharing. Although he considered different input (joist modulus of rupture) distributions he did not compare the resulting distribution with respect to the effect of variations in input.

Machine rating of lumber has not gained the use expected in the early 1960's. Kennedy (6) recommends reconsideration of machine rating lumber. He contends that, because deflection governs most design, modulus of elasticity is a more significant parameter than modulus of rupture. Machine rating lumber yields modulus of elasticity and thus has particular value to designers.

Several investigators (7, 8) have reported on the correlation of modulus of elasticity and modulus of rupture. This work contributes to the ability of applying strength criteria in design with lumber which has been graded in terms of modulus of elasticity.

A review of current concepts concerning analysis of wood joist floor systems along with methods of simulation for use with mathematical models can be found in appropriate sections of this thesis.

CHAPTER II

SIMULATION TECHNIQUE AND FLOOR DEFLECTION MODELS

2.1 Exact Derivation of Deflection Probability Distributions

The floor deflection models predict deflection based upon the floor geometry and member sizes, material and connector properties, and applied loads. In general, if the input variables of a function are random in nature, the resulting output will also be random. For wood floors values of floor geometry, material and connector properties, and applied loads all occur at random from population probability distributions. Substitution of these random floor parameters into a mathematical floor model yields variations in the floor deflection.

The distribution (9) of the dependent variable of a function based upon random independent variables of given probability distributions can be derived through exact analytical techniques for many types of functions and input probability distributions. Let Y represent an arbitrary function of the independent variables, X_i ($Y = g(X_1, X_2, \dots, X_n)$). The problem is to determine the probability distribution of Y , $F_Y(y)$, based upon known probability density distributions, $f_{X_i}(x_i)$, of the random input variables.

The probability (9) distribution of Y , $F_Y(y)$, can be written as

$$F_Y(y) = P(Y \leq y) = P(g(X_1, X_2, X_3, \dots) \leq y)$$

where $P(i \leq j)$ indicates the probability of i being less than or equal to j . The probability $F_Y(y)$ can be evaluated by integration of $f_{X_i}(x_i)$ over limits defining the ranges of x_i .

$$F_Y(y) = \int_B f_{X_1 X_2 X_3 \dots X_n}(x_1 x_2 x_3 \dots x_n) dx_1 \dots dx_n$$

where

$$B = \{(x_1, \dots, x_n) \in R_n; g(x_1, \dots, x_n) \leq y\}$$

and

R_n is n -dimensional space.

The symbol \in implies that (x_1, \dots, x_n) is contained in R_n .

Application of this technique is difficult if the functional form of $y = g(X_1, X_2, X_3, \dots, X_n)$ is complicated. This is because of the difficulty in evaluation of the integral representing the probability distribution of Y , usually expressed as an iterated integral. In the case of the floor models, deflection is not expressed in a functional form of the joist modulus, for example, but rather as a numerical approximation. Thus for this case a direct analytical approach becomes extremely complicated.

2.2 Monte Carlo Simulation of Deflection Distribution

The complexity of deriving a probability distribution of floor maximum deflections through a direct analytical process indicated a simulation technique would probably be more effective. For that reason, the Monte Carlo technique was examined.

The concept of Monte Carlo analysis (10), as applied to probabilistic problems, is the experimental observation of random phenomena, such as property or dimension values, and the inference of a solution to a problem from the behavior of systems composed of these random values. The random values used in the Monte Carlo analysis are generated in a manner that is consistent with the physical characteristics of the process or population being simulated.

Monte Carlo analysis (10) is powerful when complemented with partial theoretical formulation of the problem. Variance reducing techniques, such as stratified sampling, importance sampling, or the use of control

variates, serve to increase the simulation efficiency by restricting sampling to more representative values, and thereby, reducing variation in the output distribution. These techniques generally rely upon replacement of part of the simulation with exact theoretical analysis.

Shinozuka and others (11) have applied the Monte Carlo technique to a wide variety of structural dynamics and related problems. The basic solution technique is the random simulation of structural loading combined with the numerical solution of system governing equations. The analysis yields system response for the assumed loading function.

In this simulation of wood joist floor systems under uniformly applied loading, the behavior of maximum deflection was observed with respect to random variations in joist modulus of elasticity (MOE). Simulations were conducted to determine changes in deflection behavior for floors composed of joists generated from one MOE distribution as compared to the deflection behavior of floors with joists of different distributions. The probability distributions of MOE utilized in the random sampling represented in-grade populations of joist properties, having equal means but different standard deviations. The effect of changes in variability of joist MOE probability distributions (changes in coefficient of variation) upon floor maximum deflection populations was examined by means of Monte Carlo simulations. One hundred floors were analyzed for each degree of joist MOE variability under consideration. In the simulation, the three MOE population distributions examined represented high, medium and low variability, exhibiting coefficients of variations of approximately 0.4, 0.2, and 0.05, respectively.

Each simulation was initiated with the selection of typical floor dimensions and material and connector properties (with the exception of

joist MOE) (Figure 2.1). Floor joist MOE values were generated with respect to the low variability probability distribution and assigned to joists in the order generated. The simulated floors were analyzed by a mathematical model for prediction of floor deflections, and the maximum deflection of each floor recorded.

Floor joist MOE values were then generated for medium and high variability probability distributions and the corresponding floor deflections computed in the same manner as described in the previous paragraph.

Probability distributions for the maximum deflection of the floors for each degree of variability of the joist MOE were approximated from the maximum deflections recorded during the simulations. Characteristics of the distributions were compared in terms of differences in means, standard deviations, and coefficients of variation. Conclusions were drawn concerning the potential of increasing efficiency through application of the results of the simulations.

Numerical approximation methods of the floor deflection models and the multivariate nature of the simulation discouraged application of variance reducing techniques.

2.3 Floor Deflection Models

The mathematical model of floor deflection behavior has been developed under other portions of this research effort (12). One version employs a finite element technique for solution; the other utilizes a finite difference approximation. The floor is modeled as a system of crossing beams, T-beams in the direction parallel to the joists and sheathing strips perpendicular to the joists (Figure 2.2). Each beam is analyzed as a layered beam with incomplete composite action due to inter-layer slip. The crossing beam model neglects the contribution of

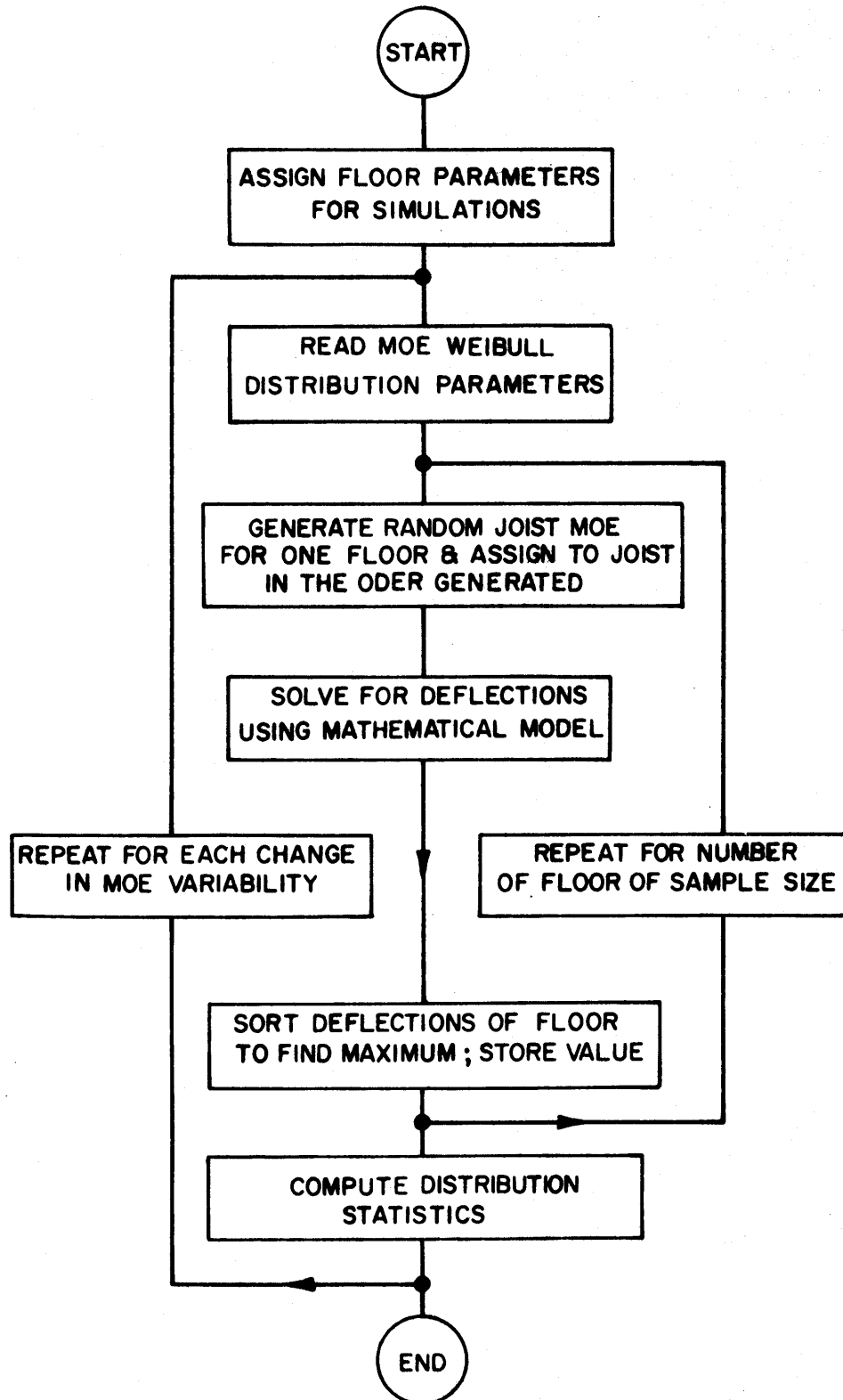


Figure 2.1
SIMULATION PROCESS DIAGRAM

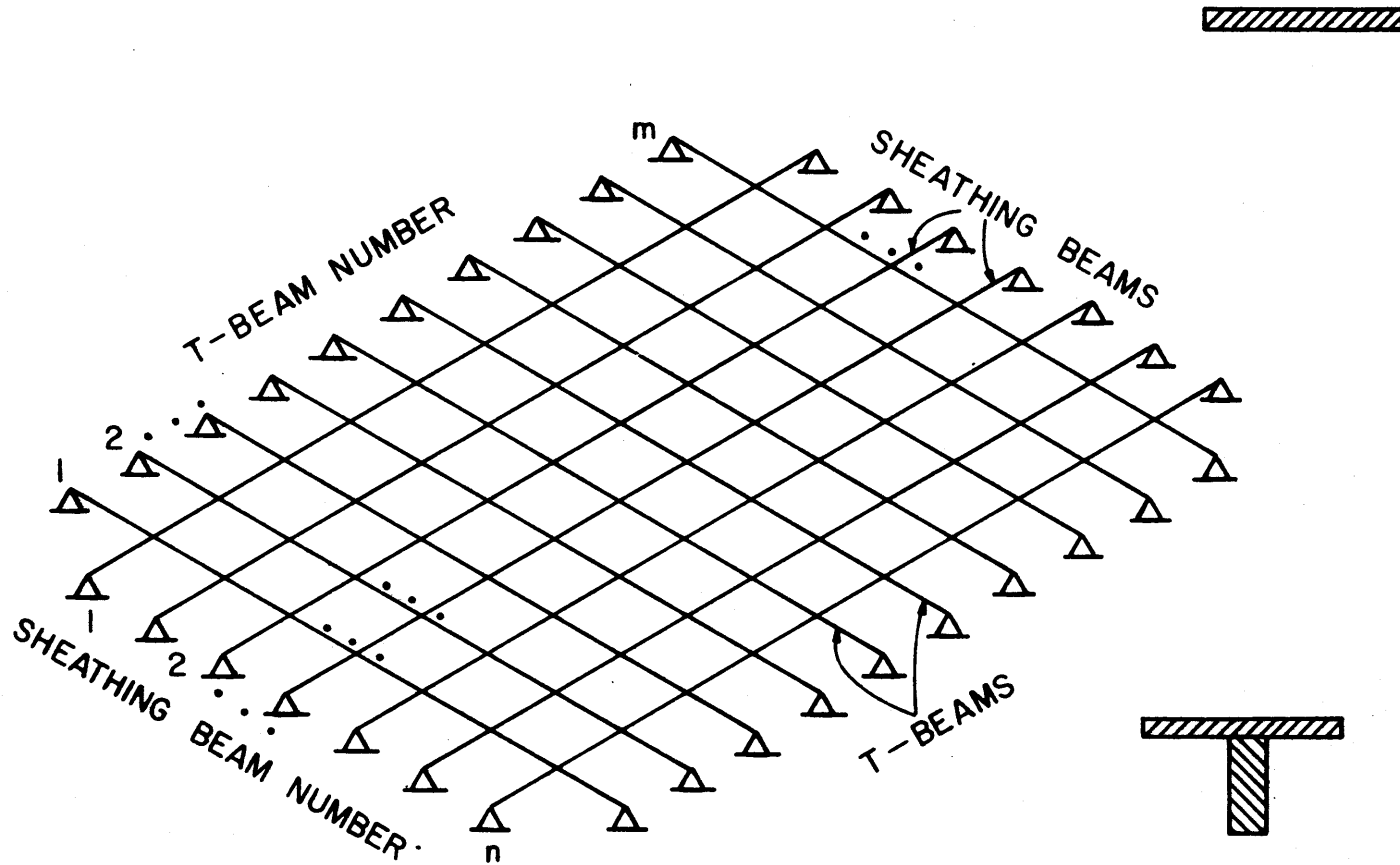


Figure 2.2
IDEALIZATION OF FLOOR SYSTEM

torsional stiffness to the total floor rigidity. Liu (13) reported in detail on the model's theoretical foundations and experimental verification. For full scale test floors representing a wide variety of floor configurations, Liu indicated that the finite element solution technique predicts deflections usually within 6 to 7 percent of the experimental observation. He further indicated that the finite difference approximation yielded deflection predictions usually within 10 to 12 percent of experimental observation. Liu indicated that the closer agreement of the finite element technique was due to its ability to include the effects of gaps in the sheathing layer.

The mathematical models of floor deflection behavior are based on the general theory of layered beams with interlayer slip developed by Goodman (14) and extended by Ko (15) (Figure 2.3). The governing equations of a layered beam when specialized to the case with one axis of symmetry are

$$\frac{d^2 y}{dx^2} = \frac{-M_t + (C_{ij} F)}{EI_i} \quad (1)$$

$$\left(\frac{s}{kn}\right) \frac{d^2 F_i}{dx^2} = \left(\frac{1}{EA_i^*} + \frac{1}{EA_j^*}\right) F_i + C_{ij} \frac{d^2 y}{dx^2} \quad (2)$$

where

E_i = MOE of ith layer,

I_i = moment of inertia of ith layer,

A_i = area of ith layer,

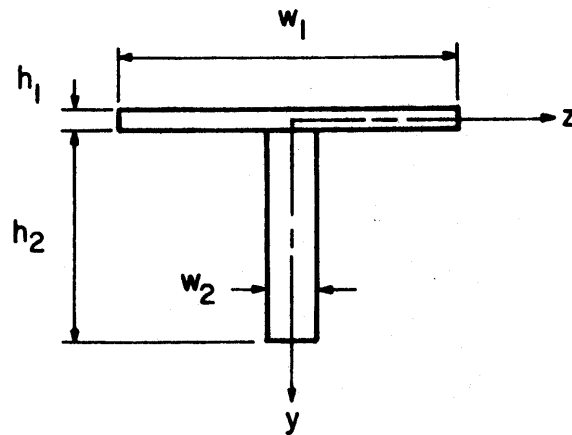
s = connector spacing,

k = slip modulus relating interlayer slip and load,

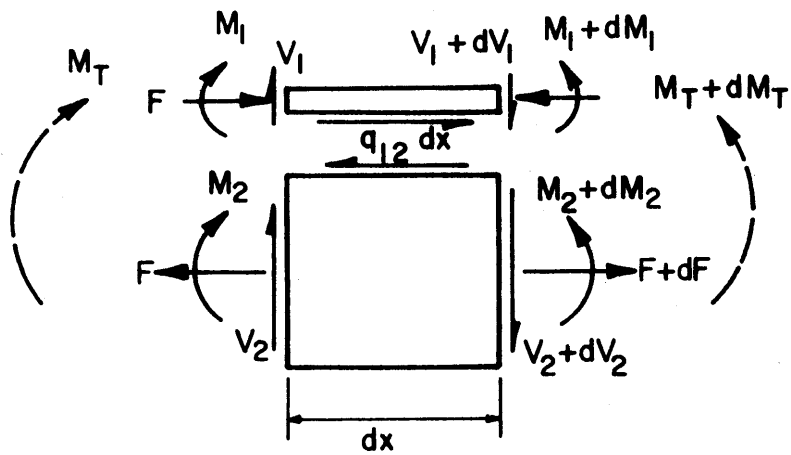
n = number of rows of connectors,

M_t = moment of cross section, and

$C_{ij} = (h_i + h_j) / 2.$



(a) CROSS SECTIONAL DIMENSIONS



(b) FORCE COMPONENTS OF A BEAM ELEMENT

Figure 2.3
LAYERED BEAM SYSTEM

The assumptions necessary in the formulation of these layered beam governing equations are

1. small deflection,
2. linear elastic materials,
3. linear variations of strains over the depth of each layer,
4. linear slip modulus,
5. negligible shear deformations,
6. equal curvature of each layer during bending.

Kuo (16) conducted a thorough investigation of the layered beam using a mathematical model including interlayer slip and gap effects. He compared experimental results of full-scale layered T-beams with the predictions of the mathematical models for beams corresponding to those tested. The T-beams used in the verification varied in the amount of composite action provided. Discontinuities in the T-beam sheathing (occurring in the test T-beams at joist locations where one piece of sheathing ended and another began) were approximated by allowing the approximating function for axial displacement to be discontinuous at the gap location. Kuo found that the finite element layered beam model closely predicts the load-deflection behavior of layered beams over a wide range of beam configurations. He indicated that predictions of deflection for beams loaded in the usual range of working loads compared closely to experimental observation.

The solution of the floor deflection model using the finite difference approximation is developed with the aid of matrix theory (12). Deflections of the T-beam can be written in terms of the T-beam flexibility matrix and the loads applied to the T-beam. Finite difference approximations for the derivatives of deflections were substituted into

the governing equations for layered beams with interlayer slip. A set of simultaneous equations of beam deflection result from application of the governing equations to each of the nodal points representing the T-beam for all T-beams in the floor.

$$\{DT\} = [FT] \{AT\} \quad (3)$$

where

$\{DT\}$ = T-beam nodal point deflections,

$[FT]$ = T-beam flexibility,

$\{AT\}$ = external loads applied to T-beam at nodal points.

A similar set of equations specifying sheathing strip response is

$$\{DS\} = [FS] \{AS\} \quad (4)$$

where

$\{DS\}$ = sheathing nodal point deflections,

$[FS]$ = sheathing flexibilities,

$\{AS\}$ = external load applied to sheathing at nodal points.

Equilibrium requires that the nodal point loads of the sheathing strips and T-beams sum to the total applied nodal loads at each nodal point.

$$\{A\} = \{AS\} + \{AT\} \quad (5)$$

where

$\{A\}$ = total externally applied nodal loads.

Compatibility requires that the nodal point deflections of sheathing and T-beams be equal.

$$\{D\} = \{DS\} = \{DT\} \quad (6)$$

Combination of these equations yields

$$\{AT\} = [FT + FS^{-1}] [FS] * \{A\} \quad (7)$$

with $\{AT\}$ known the deflections can be evaluated from the equation relating T-beam deflection to T-beam nodal point loads.

The finite element approximation solves for floor deflections by means of a Rayleigh-Ritz procedure (13). The potential energy of each beam is computed and the energies of all beams are combined. The potential energy of a layered beam with interlayer slip consists of energy due to

1. pure bending of each layer,
2. axial elongation of each layer,
3. interlayer slip of connectors between each layer, and
4. external loading.

Equilibrium of the system requires that the potential energy of the system be a stationary value. This is equivalent to determining the floor configuration at which the first variation of potential energy is zero.

In the finite element approximation, the potential energy of the beams comprising the floor is written in terms of the nodal point deflections and slopes as well as the axial displacements of each layer. A governing set of algebraic equations for the above unknown variables is found by setting the variation of the potential energy equal to zero.

A comparison of the deflections predicted by solution of the mathematical model by the finite difference and finite element techniques is given in Table 2.1. The deflections shown are for floors with 0.75 inch sheathing, slip modulus of 30,000 lb/in, uniform joist MOE of 1,800,000 psi, and uniform loading of 40 psf. Other dimensions are the same as used in the simulations.

The deflections listed in Table 2.1 indicate the large effect of gaps in the sheathing. Simulation III assumed open gaps, and would be expected to be conservative. The finite element and finite difference techniques show good agreement for the no gaps case.

TABLE 2.1
MAXIMUM FLOOR DEFLECTIONS PREDICTED BY
DIFFERENT SOLUTION TECHNIQUES

FINITE
DIFFERENCE
W/O GAPS

0.266"

FINITE
ELEMENT
W/O GAPS

0.244"

FINITE
ELEMENT
W/OPEN GAPS

0.386"

CHAPTER III

INPUT PROPERTIES FOR SIMULATIONS

3.1 Joist Modulus of Elasticity Distributions

The effects of variability and changes in variability of joist MOE on floor system deflections were simulated by a Monte Carlo technique to evaluate these effects for in-grade variations of population probability distributions. The potential of increasing wood utilization efficiency by modification of the magnitude of a grade's variability, such as by modifying grading method techniques, was then examined from simulation results.

The joist MOE probability distributions were derived from data from several sources. The distributions are realistic representations of the possible variations of MOE within a grade, but do not represent actual data occurring from a single grade in which variations are the result of lumber grading technique differences. The MOE distributions incorporated into the simulation exhibit the same mean value (1.8×10^6) and have coefficients of variations of approximately 0.056, 0.204, and 0.408. Although the distributions are not derived from actual data of a specified species and grade, they are representative of the distribution shapes and variability occurring in actual grades.

The narrow (or low variability) distribution was taken from the data presented in a paper by Galligan and Snodgrass (2) (Figure 3.1). The data, presented in the form of a histogram, showed MOE values of a population of joists resulting from machine grading of 1.8E-2100f lumber. The mean MOE for the 219 specimen sample is 1.8×10^6 psi while the coefficient of variation is 0.056.

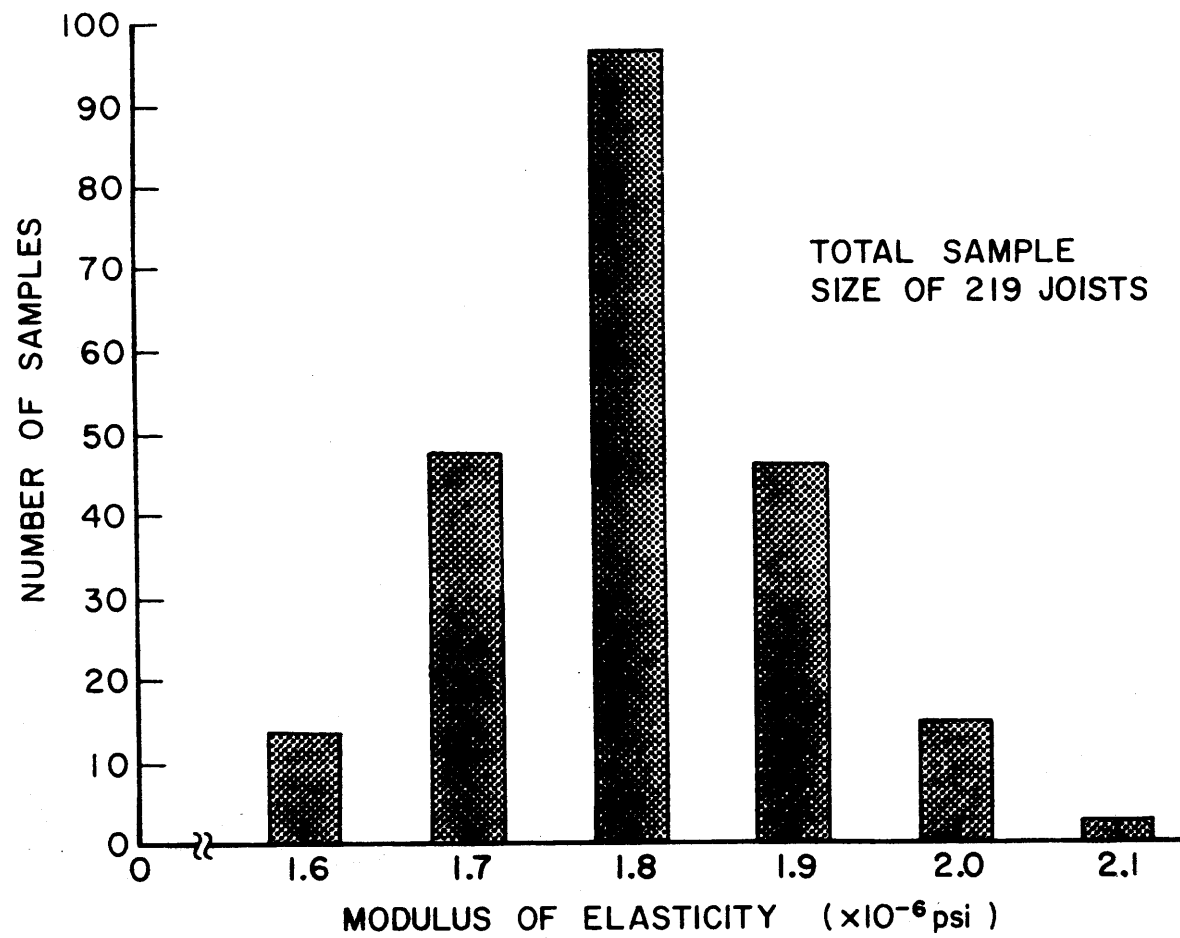


Figure 3.1
MOE DISTRIBUTION FOR 1.8E-2100f JOISTS

The medium (or medium variability) distribution was derived from data of joist MOE of Douglas-fir joists measured by the Wood Science Laboratory at Colorado State University as a part of the research project on wood joist floor systems (Figure 3.2). The data represents 90 pieces of lumber from a single visual grade. The mean MOE is 1.707×10^6 psi and the coefficient of variation is 0.215. The distribution was translated to a mean MOE value of 1.8×10^6 psi with a coefficient of variation of 0.204. This was done so that only the variability of the distribution existed as the simulation variable, and the effects of different means would not enter.

The wide (or high variability) distribution was derived by modifying the medium variability distribution. The amount each data point varied from the mean was increased by a constant factor. This maintained the same mean MOE, but increased the coefficient of variation to 0.408.

Weibull distribution functions (18) were fitted to the joist MOE distribution data by a least squares method (Figure 3.3). The Weibull distribution is a cumulative distribution as used here to describe the proportion of joists with MOE values less than given MOE values. Corresponding to any MOE value is a percentage of the total population with a value less than or equal to that modulus. Normal distribution functions were also evaluated but due to the skewness of the modulus data, the Weibull functions fit the data better. The procedure used in the least squares curve fit is outlined below.

The cumulative Weibull distribution function can be written as (5)

$$Y = 1 - e^{-\left(\frac{X - X_0}{\omega}\right)^m} \quad (8)$$

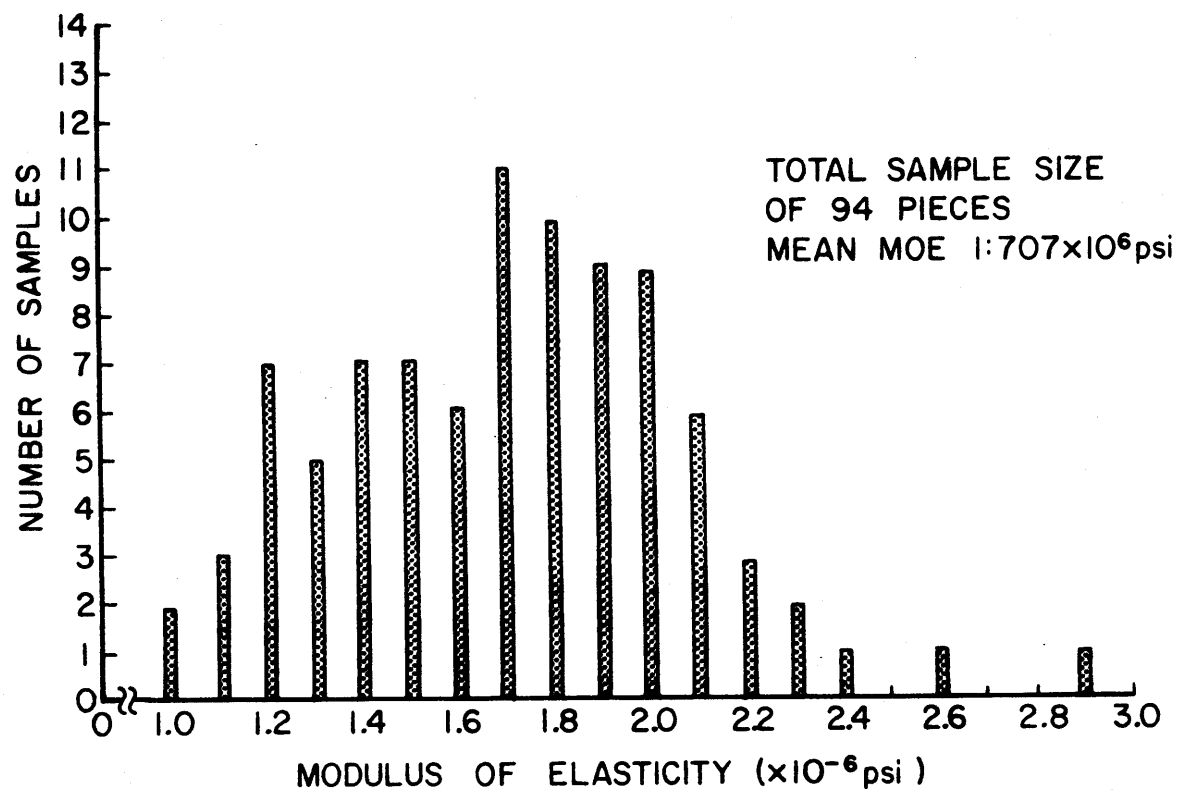


Figure 3.2
MOE DISTRIBUTION FOR VISUAL GRADE JOISTS

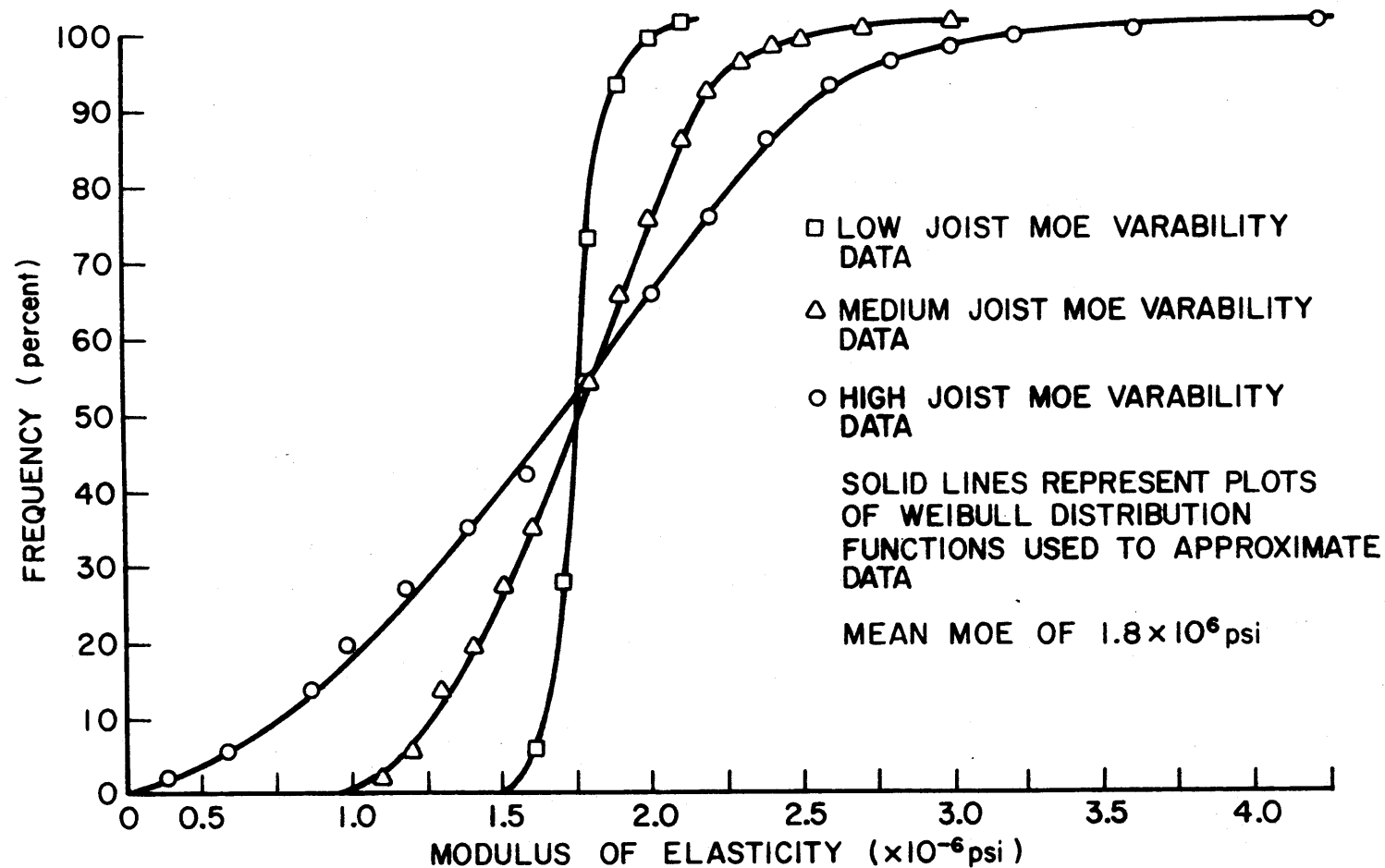


Figure 3.3
JOIST MOE CUMULATIVE DISTRIBUTIONS

This equation can be rearranged to yield

$$\ln \omega + \frac{1}{m} \ln(\ln(\frac{1}{1-y})) = \ln(X-X_0) \quad (9)$$

Let:

$$Z = \ln (X-X_0) \quad (10)$$

$$W = \ln (\ln(\frac{1}{1-y})) \quad (11)$$

$$A = \frac{1}{m} \quad (12)$$

$$B = \ln \omega \quad (13)$$

Then from Eq. (9)

$$Z = AW + B \quad (14)$$

The least squares estimates A and B were determined by a least squares technique in which the sum of the squares of the error (E) between each data point and the approximating curve at that point was minimized with respect to A and B.

The sum of squares of the error is

$$E = \sum_{i=1}^n (Z_i - AW_i - B)^2 \quad (15)$$

Differentiating with respect to the coefficients A and B yields

$$\frac{\partial E}{\partial A} = \sum_{i=1}^n 2(Z_i - AW_i - B) (-W) = 0 \quad (16)$$

$$\frac{\partial E}{\partial B} = \sum_{i=1}^n 2(Z_i - AW_i - B) (-1) = 0 \quad (17)$$

Equations (16) and (17) can be rewritten in the form of normal equations (17).

A computer program computed the coefficients of A and B in the normal equations and solved the normal equations simultaneously for A

and B. X_0 , assumed known for the curve fitting, was determined by computing A and B for several values X_0 and selecting the value of X_0 which corresponded to the minimum total error. Table 3.1 shows the values of X_0 tried, with the corresponding values of the Weibull parameters and approximation error also listed. The set of Weibull parameters used in the simulation is indicated.

Random joist MOE values were generated during the computer simulation. A system subroutine for generating random numbers between zero and one from a uniform distribution was called during the simulation to return a random number. This number represented the frequency of occurrence, Y. Substitution of the frequency into the Weibull function yielded a value of X, corresponding to the random MOE. This process was repeated for generation random modulus for each simulation floor joist. Random MOE values generated from the high variability probability distributions were limited to values between 600,000 psi to 3,000,000 psi. This was done to eliminate unrealistic values generated from the ends of Weibull distribution. Typical sets of random joist MOE values are listed in Table 3.2.

3.2 Floor Configurations for Simulation Studies

Since joist MOE was the variable in the study, other floor parameters, such as sheathing and connector properties and floor geometry, were held constant during a specific simulation. The constant floor parameters were selected such that the simulation floors would be generally typical of those currently being built for residential housing. Simulation of typical flooring systems using an accurate mathematical model provides deflection distributions from which conclusions of effects of variability on flooring systems can be drawn directly. This eliminates

TABLE 3.1

WEIBULL DISTRIBUTION CURVEFIT RESULTS

$$\text{Weibull Function: } y = 1 - e^{-\left(\frac{x-x_0}{\omega}\right)^m}$$

LOW VARIABILITY DISTRIBUTION

<u>x_0</u>	<u>ω</u>	<u>m</u>	<u>STANDARD ERROR</u>
1.52	0.262	2.270	9.1×10^{-5}
1.51	0.278	2.466	7.2×10^{-5}
1.50*	0.292	2.644	6.9×10^{-5}
1.49	0.306	2.812	7.2×10^{-5}
1.45	0.356	3.420	12.3×10^{-5}

MEDIUM VARIABILITY DISTRIBUTION

<u>x_0</u>	<u>ω</u>	<u>m</u>	<u>STANDARD ERROR</u>
0.70*	1.177	3.127	5.1×10^{-5}
0.80	1.063	2.841	6.6×10^{-5}
0.90	0.944	2.439	13.4×10^{-5}
1.00	0.813	1.976	36.7×10^{-5}
0.60	1.288	3.577	6.0×10^{-5}
0.67	1.211	3.326	5.2×10^{-5}

HIGH VARIABILITY DISTRIBUTION

<u>x_0</u>	<u>ω</u>	<u>m</u>	<u>STANDARD ERROR</u>
0.25	1.540	1.841	4.9×10^{-4}
0.10	1.749	2.223	2.4×10^{-4}
0.00	1.875	2.443	1.7×10^{-4}
-0.10	1.997	2.650	1.4×10^{-4}
-0.40	2.434	3.224	1.5×10^{-4}
-0.22*	2.138	2.885	1.3×10^{-4}

* Values used in simulations.

TABLE 3.2

SAMPLES OF RANDOM JOIST MOE

<u>BEAM</u>	<u>RANDOM NUMBER</u>	<u>LOW VARIABILITY MOE</u>	<u>MEDIUM VARIABILITY MOE</u>	<u>HIGH VARIABILITY MOE</u>
FLOOR #41				
1	.58694	1778716.	1831565.	1828691.
2	.25129	1682693.	1491722.	1171103.
3	.01081	1552813.	977232.	600000.
4	.90186	1901528.	2240786.	2642760.
5	.72462	1821484.	1976735.	2115043.
6	.42480	1733394.	1673892.	1521170.
7	.50779	1756367.	1754351.	1677614.
8	.06505	1605205.	1196491.	618876.
9	.99948	2127620.	2947817.	3000000.
10	.17024	1654758.	1386077.	974847.
11	.03505	1582779.	1105394.	600000.
FLOOR #79				
1	.3067	1699716.	1553668.	1289453.
2	.4982	1753706.	1745042.	1659557.
3	.6505	1797600.	1896057.	1955546.
4	.8293	1862245.	2112328.	2384994.
5	.5852	1778221.	1829864.	1825354.
6	.4009	1726768.	1650463.	1475816.
7	.7003	1813359.	1949396.	2060897.
8	.0253	1573089.	1064888.	600000.
9	.2748	1690090.	1518744.	1222637.
10	.3416	1709957.	1590535.	1360236.
11	.8917	1895004.	2219593.	2600105.

the need to predict the behavior of real floors from the results obtained from use of a simplified floor system or deflection model.

An eleven joist floor with 16 inch spacing between joists was selected. The joint span of 160 inches was assumed, the span allowed by Span Tables for Joists and Rafters (18) based upon 2 x 8 inch nominal joist dimensions, a mean MOE of 1.8×10^6 psi, a deflection limit of $L/360$, and a uniform load of 40 psf.

Two basic floor configurations were simulated in the study. The first floor consisted of one layer of sheathing of 0.75 inch thickness, connected to the joists by 8 penny nails at 8 inch spacing. A slip modulus of 30,000 lb/in was used, based upon the observed composite behavior for this type of connection as reported by Kuo (15). A second floor configuration, used to examine floors of very minimum construction, assumed the use of 0.5 inch sheathing and a slip modulus of 15,000 lb/in.

MOE values assumed for both 0.5 inch and 0.75 inch sheathing were average values for Douglas fir plywood, as measured by the Wood Science Lab at Colorado State University. MOE values for the 0.75 inch sheathing are 523,000. psi and 1,361,000. psi, perpendicular and parallel to the surface grain, respectively. MOE values of the 0.5 inch sheathing are 245,000. psi and 1,645,000 psi, perpendicular and parallel to the surface grain, respectively. The MOE values perpendicular to the surface grain represent gross cross section values adjusted to compensate for the difference in MOE for bending and axial loads, as explained by Liu (12).

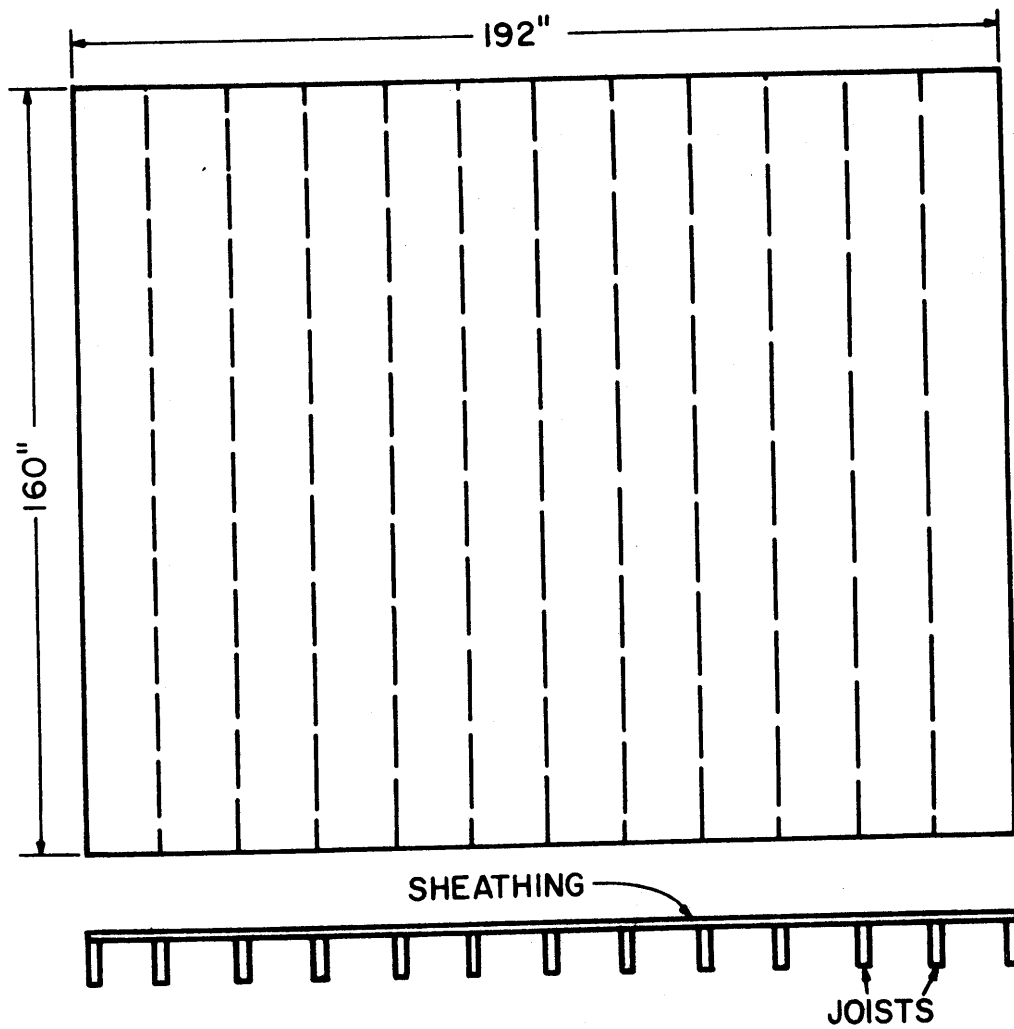
Gaps in the sheathing were considered for the simulations conducted in which the mathematical floor model was solved by the finite element technique. Gaps were included by allowing the approximating function for axial displacement to be discontinuous at the gap location and

physically modeled an open gap situation. The use of flexible gaps approximating tightly butted tongue and groove or glued gaps would result in somewhat stiffer floors.

A uniform live load of 40 psf was applied to each simulation floor. This loading corresponds to the design loading on which the deflection limit of $L/360$ of the span tables is founded.

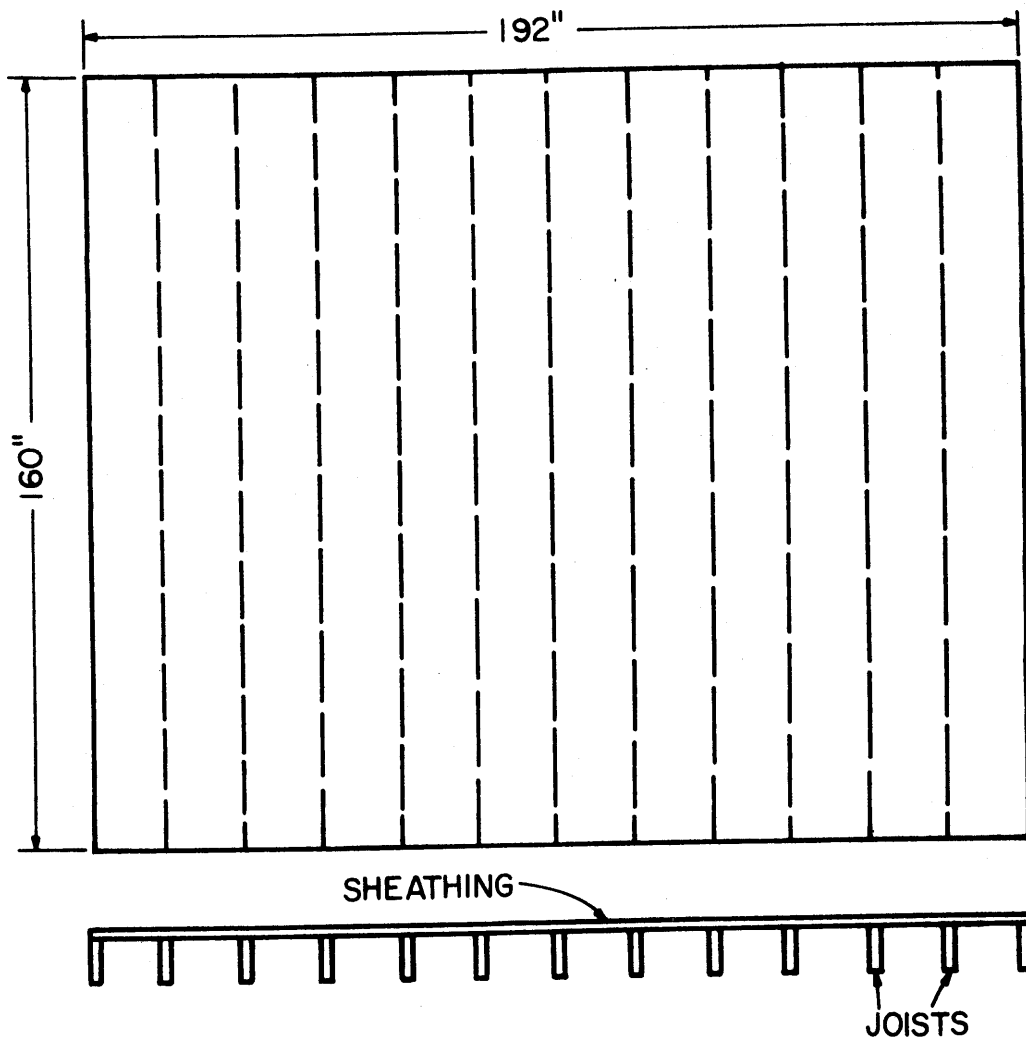
Schematic diagrams of each floor configuration used in the simulations are shown in Figures 3.4, 3.5, and 3.6. Also, material and connector properties are listed with the diagrams.

Simulations of these floors were conducted using joist MOE randomly generated from the Weibull MOE functions described earlier. Distributions of maximum floor deflection resulted from the simulations. These results are discussed in Chapter IV.



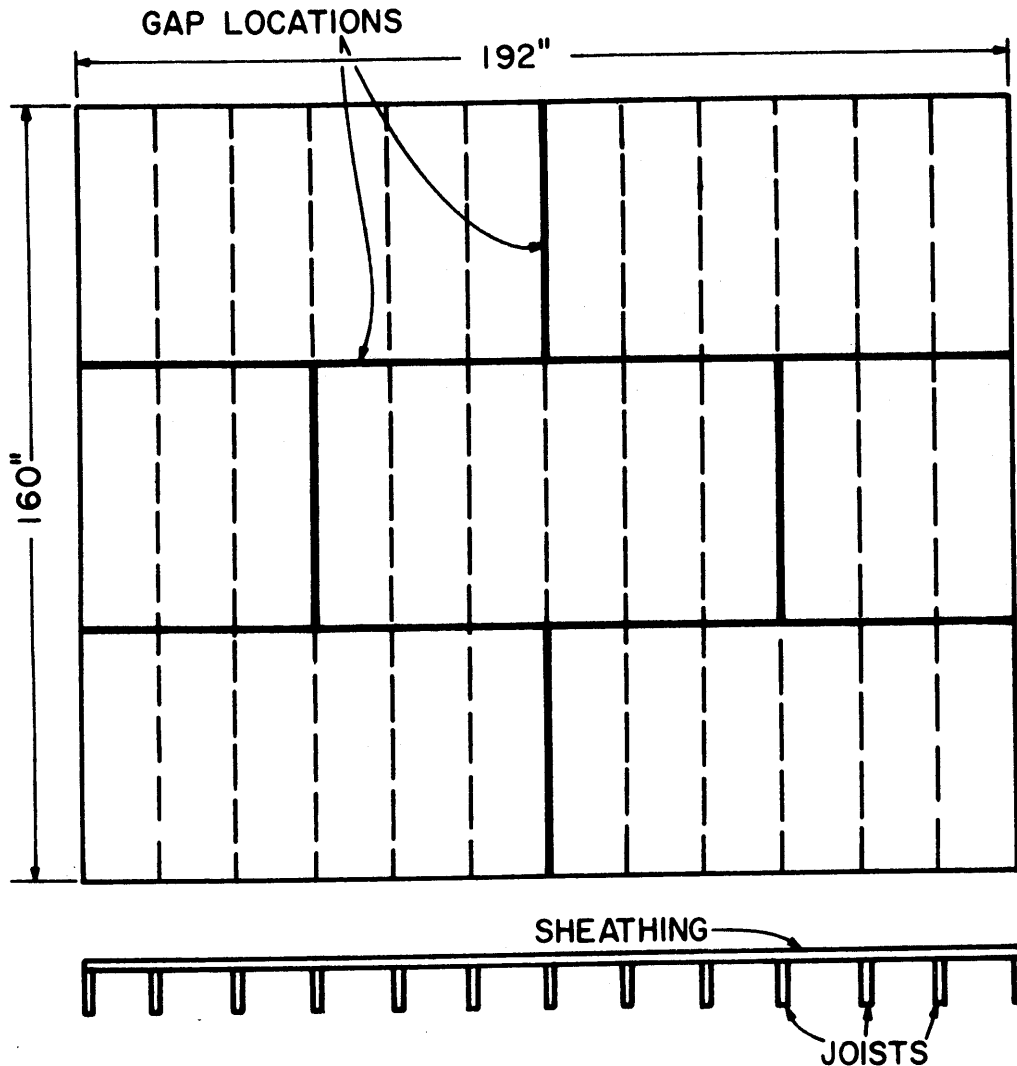
NOMINAL JOIST DIMENSION = 2x8 in.
 SHEATHING THICKNESS = 0.75 in.
 SLIP MODULUS = 30,000 lb/in.
 NAIL SPACING = 8 in.
 SHEATHING MOE
 || TO SURFACE GRAIN = 1,361,000 psi
 ⊥ TO SURFACE GRAIN = 523,000 psi
 UNIFORM LOAD = 40 psf

Figure 3.4
 SIMULATION I FLOOR CONFIGURATION



NOMINAL JOIST DIMENSION = 2 x 8 in.
 SHEATHING THICKNESS = 0.5 in.
 SLIP MODULUS = 15,000 lb/in.
 NAIL SPACING = 8 in.
 SHEATHING MOE
 || TO SURFACE GRAIN = 1,645,000 psi
 ⊥ TO SURFACE GRAIN = 245,000 psi
 UNIFORM LOAD = 40 psf

Figure 3.5
 SIMULATION II FLOOR CONFIGURATION



NOMINAL JOIST DIMENSION = 2 × 8 in.

SHEATHING THICKNESS = 0.75 in.

SLIP MODULUS = 30,000 lb/in

NAIL SPACING = 8 in

SHEATHING MOE

|| TO SURFACE GRAIN = 1,361,000 psi

⊥ TO SURFACE GRAIN = 523,000*psi

UNIFORM LOAD = 40 psf

OPEN GAPS

* TRANSFORMED CROSS SECTION

Figure 3.6

SIMULATION III FLOOR CONFIGURATION

CHAPTER IV

PRESENTATION AND DISCUSSION OF SIMULATION RESULTS

4.1 Deflection Probability Distributions

Results of the deflection simulations for the various floor configuration and joist MOE variability combinations are listed in Table 4.1. Simulation I was conducted using a crossing beam floor model without gaps and deflections were calculated using a finite difference approximation for floors with sheathing thickness of 0.75 inch and a slip modulus of 30,000 lb/in (Figure 3.4). A total of three hundred floors (one hundred for each degree of variability of the joist MOE distribution), were randomly generated and analyzed for deflections corresponding to a 40 psf uniform load during the simulation. The resulting cumulative maximum deflection distributions are shown in Figure 4.1.

Simulation II was also conducted using the crossing beam floor model assuming no gaps solved by a finite difference technique. The floors had 0.5 inch sheathing and a slip modulus of 15,000 lb/in, as indicated in Figure 3.5. One hundred floors were generated from each of the three joist MOE distributions and analyzed for a uniform loading of 40 psf. Maximum deflection cumulative probability distributions are shown in Figure 4.2.

Simulation III utilized the finite element technique for solution of the crossing beam floor model. The floors consisted of 0.75 inch sheathing with a slip modulus of 30,000 lb/in. Open gaps in the sheathing strip as indicated in Figure 3.6 were included. The maximum deflection cumulative probability distributions are shown in Figure 4.3 for deflections resulting from a uniform load of 40 psf.

TABLE 4.1
SIMULATION RESULTS

<u>SIMULATION</u>	<u>VARIABILITY</u>	<u>MEAN DEFLECTION</u>	<u>STANDARD DEVIATION</u>	<u>COEFFICIENT OF VARIATION</u>
I	Low	.275	.0045	.016
I	Medium	.296	.0177	.060
I	High	.345	.0401	.116
II	Low	.335	.0060	.018
II	Medium	.374	.0242	.065
II	High	.454	.0566	.125
III	Low	.399	.0085	.021
III	Medium	.436	.0336	.077
III	High	.519	.0793	.153

Joist MOE Coefficient of Variation

Low Variability	0.056
Medium Variability	0.204
High Variability	0.408

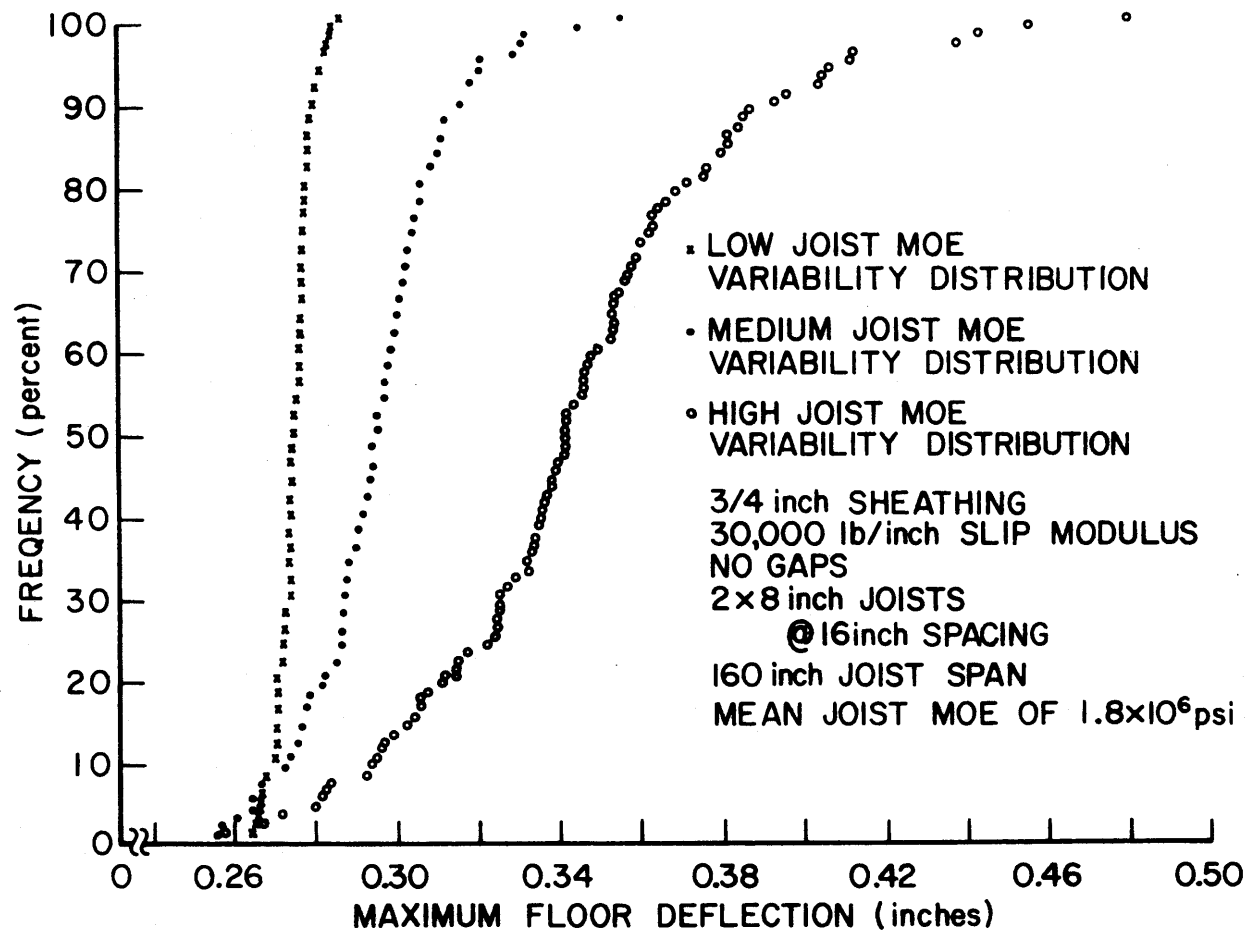


Figure 4.1
 MAXIMUM DEFLECTION DISTRIBUTION - SIMULATION I

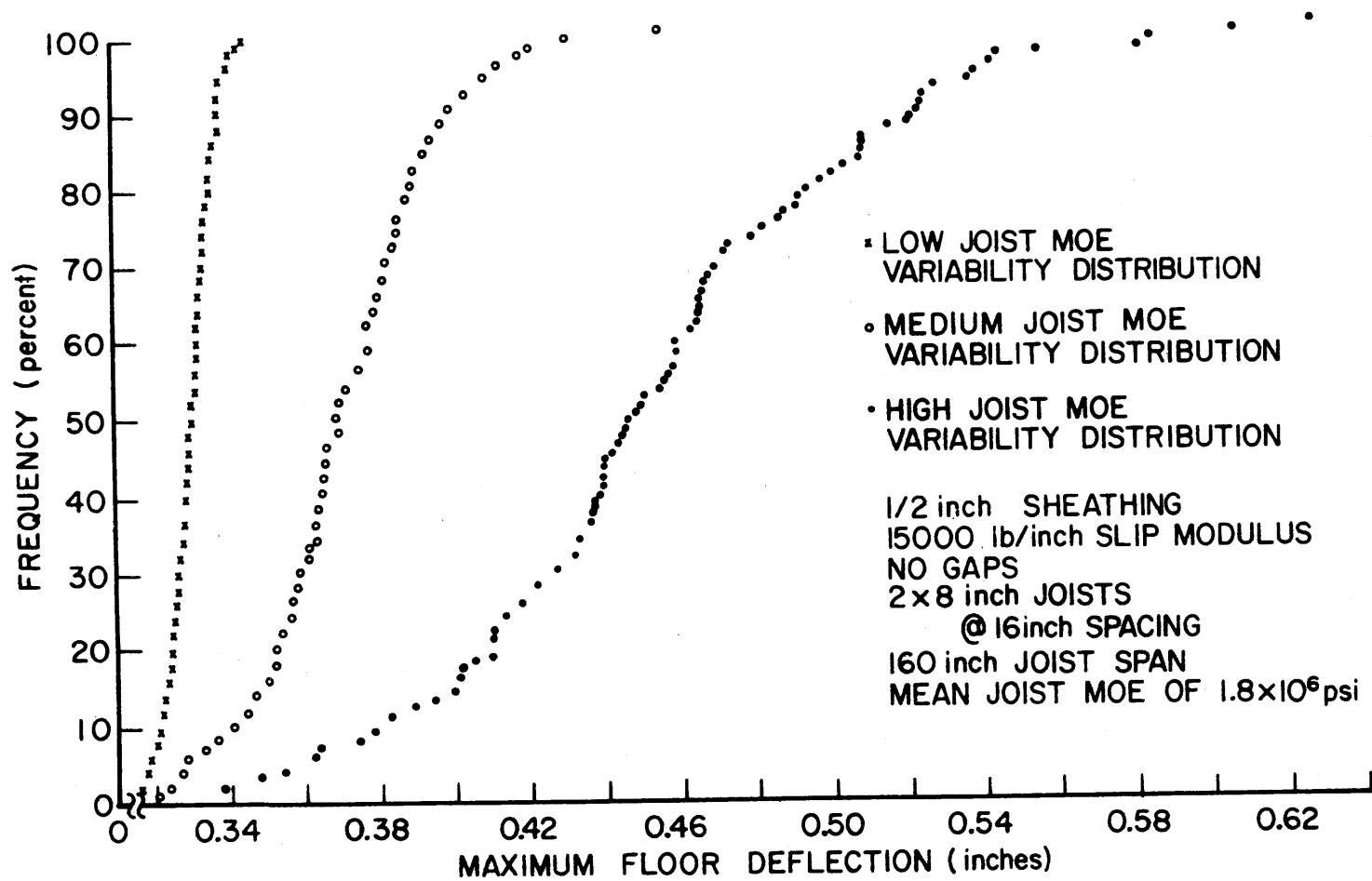


Figure 4.2
 MAXIMUM DEFLECTION DISTRIBUTION - SIMULATION II

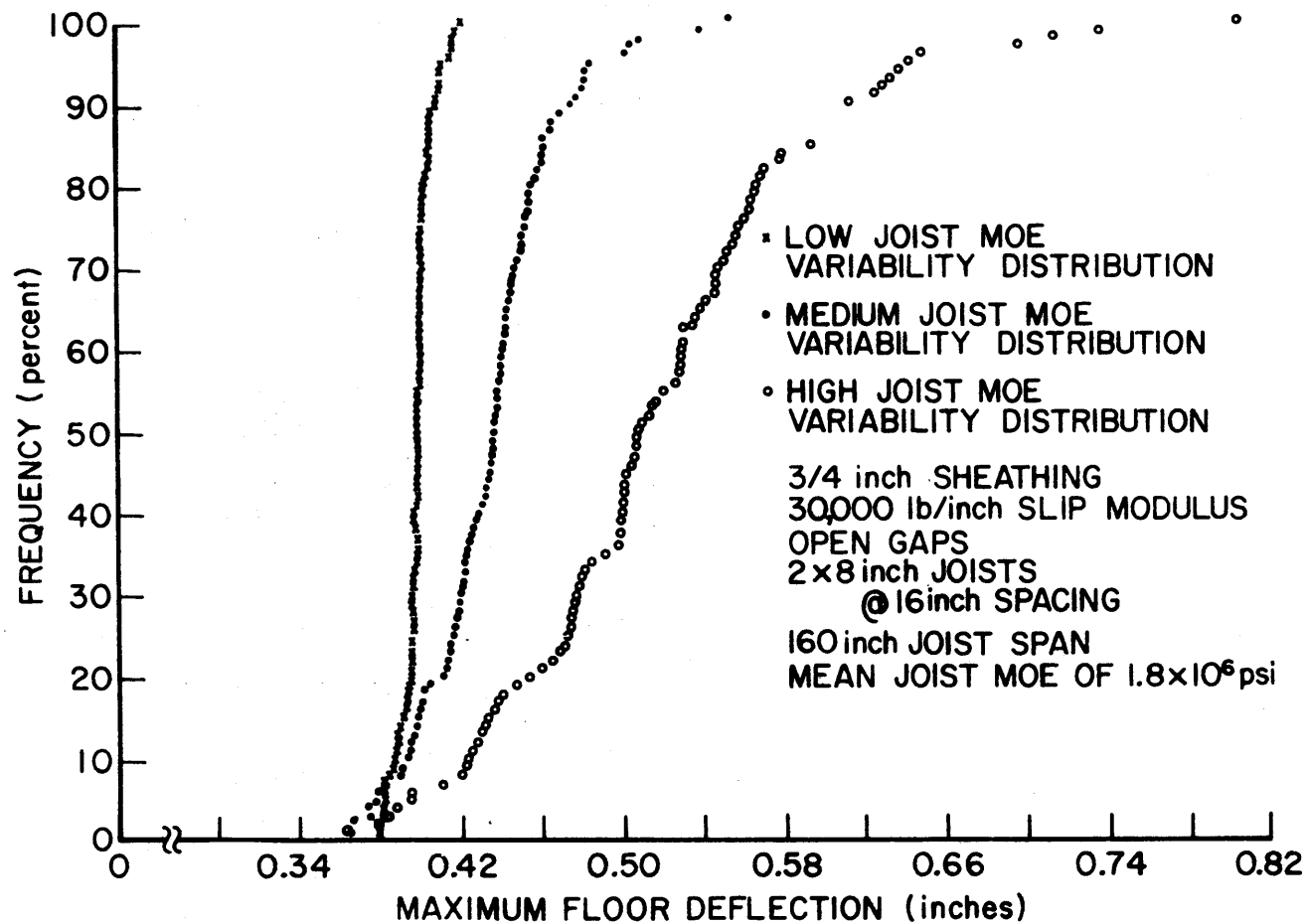


Figure 4.3
 MAXIMUM DEFLECTION DISTRIBUTION - SIMULATION III

The mean, standard deviation, and coefficient of variation were computed for each maximum deflection probability distributions. These values are given in Table 4.1.

It was observed from the maximum deflection distributions that the number of floors in each study required to yield a smooth deflection curve increased with greater joist MOE variability. For further simulation studies it is recommended that the number of floors be reduced when low variability distributions are considered and increased for high variability distributions to make more efficient use of computer time.

4.2 Discussion of Simulation Results

Several characteristics of the deflection cumulative probability distributions presented in Section 4.1 merit further discussion.

The mean maximum deflection of the deflection distribution increases as expected, with greater joist MOE variability. Mean values from Table 4.1 are plotted in Figure 4.4 showing the relationship between mean maximum deflections and joist MOE variability for the three simulations. This graph indicates that mean maximum deflection increases non-linearly as MOE variability rises.

Difference in the mean maximum deflection of floors due to variations in the floor configurations are also evident from Figure 4.4. Increased composite action as a result of utilizing thicker sheathing or providing a higher slip modulus yields a resulting floor with a stiffer overall response. The presence of open gaps in the sheathing has a strong influence on floor response. Mean maximum deflection of floors without gaps decreased approximately one-third from the mean maximum deflection of floors identical in configuration but with open gaps. This increase in mean maximum deflection demonstrates that composite

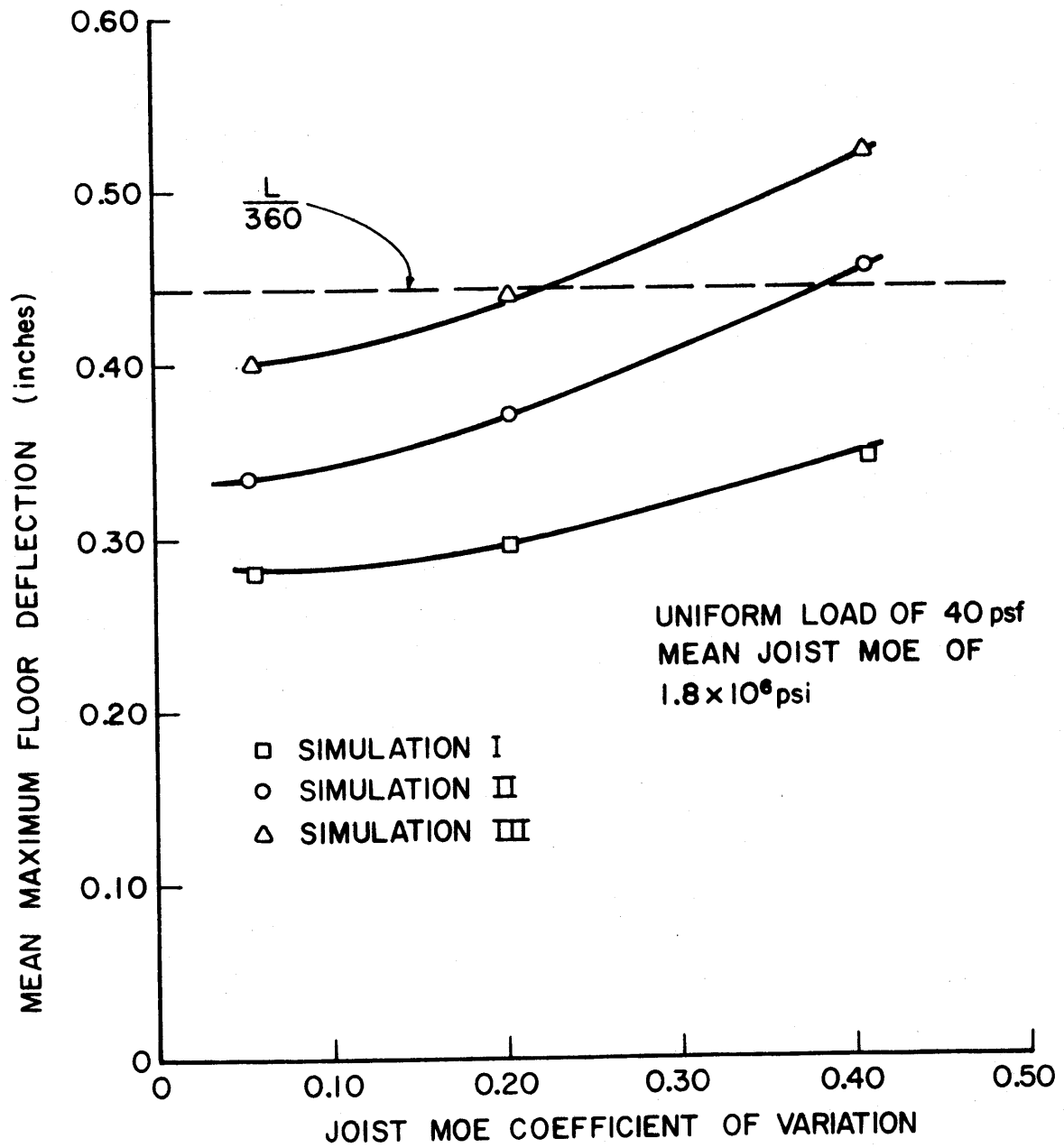


Figure 4.4
MEAN MAXIMUM DEFLECTION VS. JOIST MOE COEFFICIENT OF VARIATION

action and load sharing are substantially diminished by the presence of open discontinuities in the sheathing. Tongue-and-groove tightly butted, or glued joints do not diminish the composite action and load sharing effects as greatly as do open gaps (See Table 2.1). Thus, attention given to the details of construction can be expected to produce beneficial results.

The deflections of floors corresponding to the configurations used in the three simulations were determined for joists of uniform 1.8×10^6 psi MOE. The purpose of this was to separate the effects of structural interaction and joist MOE variability on floor deflection. From Table 4.2, it can be observed that the presence of structural interaction markedly stiffens the floor in comparison to a joist only prediction. As greater amounts of joist MOE variability are introduced, the average stiffness of a floor population decreases. For the simulations conducted only the mean maximum deflection of floor populations derived from the high variability joist MOE distribution exceeded the $L/360$ deflection criterion. Thus, the influence of grading, i.e., reducing the joist MOE variability, is clearly evident.

Another result obtained by examining the floor deflection distributions is that an increase occurs in variability of the deflection distributions with increasing MOE variability. The coefficient of variation of the maximum deflection distributions are plotted with respect to the corresponding values of joist MOE coefficient of variation in Figure 4.5. The relationships between coefficient of variation and MOE coefficient of variation are nearly linear. These results have important implications in that it may be possible to utilize such a relationship in developing probability-based design concepts.

TABLE 4.2

COMPARISON OF AVERAGE FLOOR MAXIMUM PREDICTED DEFLECTIONS

SIMULATION*	JOIST ONLY PREDICTION (L/360)	FLOORS WITH NO MOE VARIABILITY	FLOORS WITH LOW JOIST MOE VARIABILITY	FLOORS WITH MEDIUM JOIST MOE VARIABILITY	FLOORS WITH HIGH JOIST MOE VARIABILITY
I	.444	.266	.274	.296	.345
II	.444	.304	.335	.374	.454
III	.444	.386	.399	.436	.519

* For assumptions made for each simulation, refer to Section 4.1.

Considering the simulation floors generated using the same MOE distribution, it can be seen that the variability of floor maximum deflection distribution is greater for floors with higher mean deflection. For example, for the medium variability distribution, the mean maximum deflection of three simulation floors as presented in Section 4.1 are 0.296 inch, 0.374 inch, and 0.436 inch. The coefficients of variations for these floors are 0.0600, 0.0648, and 0.0772, respectively. Thus, as floor systems become less stiff, they tend to show greater deflection variability for the same joist MOE variability. This is expected as less composite action and load sharing is present to offset the effects of joist MOE variability.

Thus, two basic effects on maximum deflection distributions occur as a result of joist MOE variability and changes in variability. These effects are differences in the mean maximum deflection and changes in the amount of variability of the maximum deflection distribution.

One of the most important results that can be derived from the deflection probability distribution is concerned with the increase in the efficiency of wood use that could be derived from modification of wood variability within a grade. From the deflection distributions generated in the simulations, the percentage of acceptable floors corresponding to a specific deflection limit can be seen to decline as MOE variability rises. Thus, if the floors of a given design were required to have a specific percentage of their population exhibit deflection under design load less than a specified deflection limit, those floors constructed of low MOE variability lumber would produce a greater number of acceptable floors than floors constructed of higher variability lumber. If a particular floor configuration is examined, the number of floors that

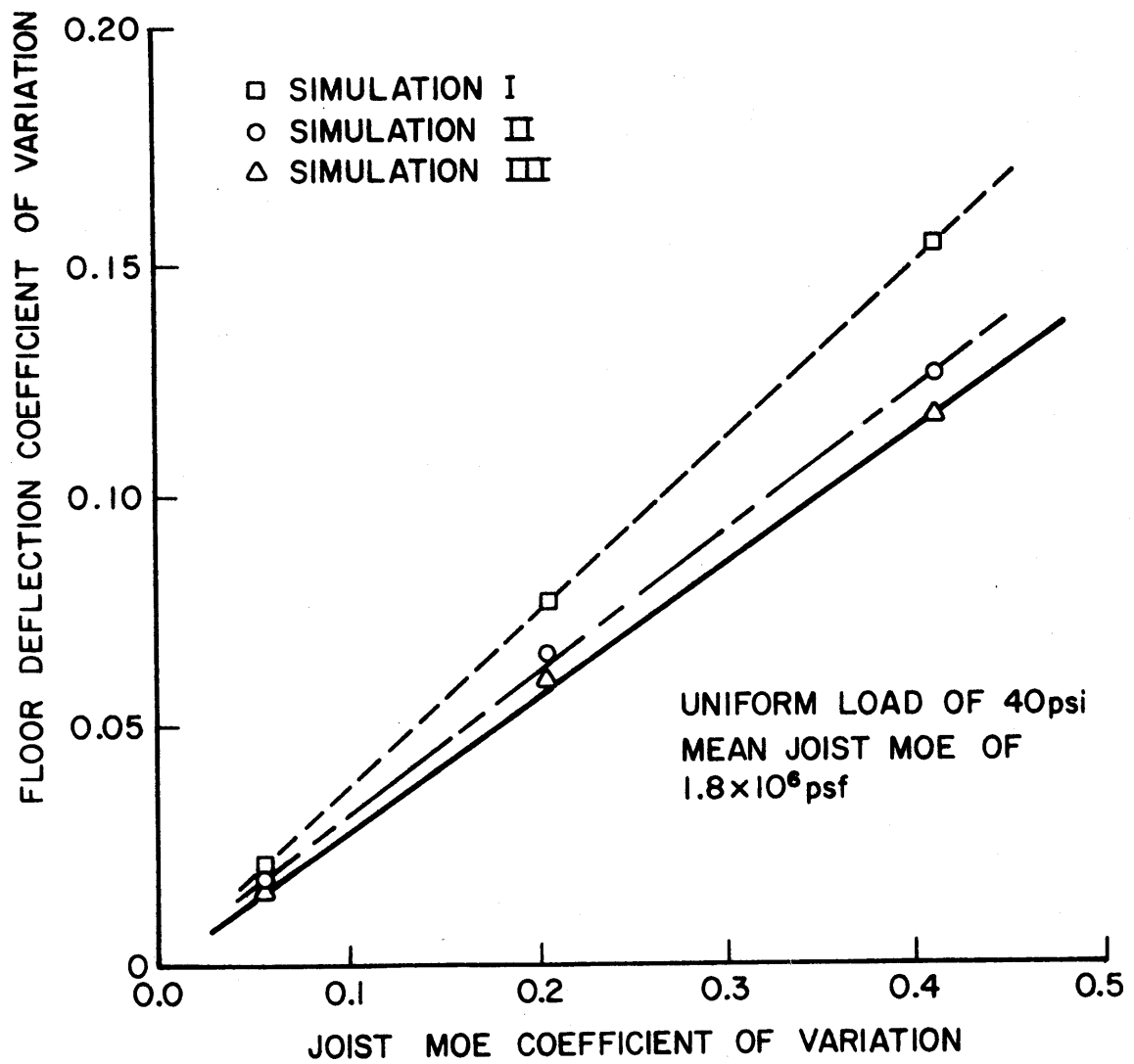


Figure 4.5

DEFLECTION COEFFICIENT OF VARIATION VS JOIST MOE COEFFICIENT VARIATION

would be acceptable, under the requirement that a certain population percentage satisfy a maximum deflection criterion, can be determined as a function of MOE variability from the maximum deflection distributions. For the simulation of floors with 0.75 inch sheathing, 30,000 lb/in, slip modulus, and open gaps, (Simulation III) all floors generated from the low variability joist MOE distribution satisfy the deflection requirement of $L/360$, while 65% of the floors generated from the medium variability joist MOE distribution satisfy the $L/360$ requirement, and only 19% of the floors generated from the high variability distribution satisfy the $L/360$ criterion. For all floors resulting from the high and medium variability joist MOE distribution to satisfy the deflection requirement, the floors would need to be modified either by increasing the mean value of joist MOE or by providing for more composite action or load sharing capability. This shows that by decreasing the joist MOE variability more restrictive deflection limits may be satisfied. Similar results are obtained from the results of Simulations I and II.

The problem can also be examined from the point of view of determining the deflection criterion that a given percentage of the floor population satisfies. For example, the maximum deflection limit that could be imposed which 95% of the floor populations of Simulation III would satisfy would be 0.412 inch, 0.483 inch, and 0.641 inch for floors generated from the low, medium, and high variability joist MOE distributions, respectively. The deflection limit corresponding to the low variability distribution shows a 16% reduction over the deflection limit from the medium variability MOE distribution. The benefit obtained from reduction of the deflection limit due to less joist MOE variability could be applied by allowing an increased load or span for this floor configuration.

Thus, the existence of component variability and degree of component interaction alters the performance of floors. Design methods which do not account for these two critical factors, are unable to properly assess the degree of satisfaction of imposed performance criteria. Thus, some floors will be overly conservative while others may be near the limit of acceptability.

One direct application of effects of joist MOE distribution variability on floor response arises with respect to lumber grading techniques. In general, less variability occurs in lumber graded mechanically than in lumber graded visually. In the simulation studies conducted, the low variability joist MOE distribution approximated a tightly selected machine graded lumber (1) and the medium variability joist MOE distribution approximated visually graded lumber. Because a greater proportion of floors can satisfy a given deflection criterion as the joist MOE variability is reduced, the potential for increased wood use efficiency through improved grading procedures is evident.

CHAPTER V

CONCLUSIONS

5.1 Conclusions

Simulation studies of the deflection behavior of wood joist floor systems were conducted. The primary purpose of the studies was to demonstrate the feasibility of using the Monte Carlo technique for evaluating the effects of component or loading variations on floor system response. When compared to the costs of studying component variability through full-scale testing, Monte Carlo analysis is very economical and very effective.

The secondary purpose of the study was the investigation of the effects of joist MOE variability and changes in joist MOE variability on the deflection response of floors. Simulation results indicate that there are two basic effects induced on deflection behavior by joist MOE variability and changes in variability. These effects are changes in mean maximum floor deflection and maximum floor deflection variability. The study includes discussion on application of design deflection limits and their relationship to the floor maximum deflection distributions. A means for seeking optimum economic efficiency through restricting joist MOE variability, or the variability of other floor components, to a value that yields the best floor maximum deflection response to joist, or other component, cost relationship can be formulated from floor maximum deflection distributions.

The study emphasizes the important roles of structural interaction and component variability on structural performance. For floors in which a deflection criterion governs design, the design calculation is normally based on the deflection behavior of joists with average member stiffness

acting alone. This method of design normally does not include the beneficial effects of load sharing and composite action nor the detrimental effects of joist MOE variability. The results indicated that design analysis based upon joist-acting-alone behavior does not necessarily describe the behavior of floors within the design, and the effects of structural interaction and component variability need to be evaluated.

The differences in the results of the three simulations conducted demonstrate the important effects sheathing, connectors, and discontinuities have on floor performance. Stiffening from structural interaction is usually adequate to offset increased flexibility due to expected joist MOE variability. With the possible exceptions of minimal floor construction or high joist MOE variability, floors designed by joist only deflection criterion are conservative. However, for an accurate prediction of floor behavior, especially when the predictions are to be used for the evaluation of economic alternatives, structural interaction and component variability must be included in the analysis.

This study dealt only with the effects of joist MOE variability on floor deflections. For more complete understanding of floor response, other floor variables and other design criterion need to be examined for their effects on floor system behavior. Factors such as slip modulus, sheathing properties, component dimensions, and the location and type of gaps exhibit variations from the nominal values used in design analysis. These variations result in floor response different from the predicted design calculations.

In the simulations conducted only a uniform load of 40 psf was considered. Concentrated loads may result in differences in deflection distributions. Simulation of loadings could be applied in the same

manner as joist MOE variability to evaluate the life time floor response to expected loading histories.

Examination of other design criterion, especially that of floor strength, could be performed by developing floor system response distribution comparable to the deflection distributions with a mathematical floor model capable of predicting floor failure. Floor strength distributions could be obtained as a function of modulus of rupture variability, for example.

From determination of the effects of component and loading variability on various design criterion, a design probabilistic method could be developed. This would allow a designer to select the degree of reliability needed for a flooring system and, from curves such as Figure 4.5, relating component variability to floor response variability, specify the components necessary.

REFERENCES

1. National Design Specification for Stress Grade Lumber and its Fastenings, 1973 Edition, recommended by National Forest Products Association.
2. Galligan, W. L., and Snodgrass, D. V., "Machine Stress Rated Lumber: Challenge to Design", Forest Products Journal Volume 20, No. 9, pp. 63-69.
3. Polensek, A., "Static and Dynamic Analysis of Wood-Joist Floors by the Finite Element Method", Ph.D. Dissertation, Oregon State University, June 1973.
4. Bonnicksen, L. W., and Suddarth, S. K. "Structural Reliability Analysis of a Wood Load-Sharing System", Journal of Materials, Vol. 1, No. 3, Sept. 1966, p. 491.
5. Zahn, J. J., "Strength of Multiple-Member Structures", USDA Forest Service Research Paper, FPL 139, July 1970.
6. Kennedy, D. E., "A New Look at Mechanical Lumber Grading", Forest Products Journal, Vol. 19, No. 6, June 1969, pp. 41-44.
7. Hulbrand, H. C., Miller, D. G., "Machine Grading - Theory and Practice, Parts I and II", Forest Products Journal, Vol. 16, No. 11, pp. 28-34, and Vol. 16, No. 12, pp. 36-40.
8. Ethington, R. L., "Some Stress-Grading Criteria and Methods of Grade Selection for Dimension Lumber", USDA Forest Products Research Paper 148.
9. Blum, J. R. and Rosenblatt, J. I., Probability and Statistics, W. B. Saunders Co., Philadelphia, 1972.
10. Hammersley, J. M. and Handscomb, D. C., Monte Carlo Methods, John Wiley & Sons, Inc., New York, N. Y., 1964.
11. Shinozuka, M. "Monte Carlo Solution of Structural Dynamics" Technical Report 19, National Science Foundations, Grant No. NSF GK 24925.
12. Vanderbilt, M. D. and Goodman, J. R. "Service and Overload Behavior of Wood Joist Floors Systems," Paper Presented to ASCE Structural Engineering Meeting, April 9-13, 1973, San Francisco, California.
13. Liu, J. S., "Verification of a Mathematical Model for Wood Joist Floor Systems", Ph.D. Dissertation, Colorado State University, Fort Collins, CO, July 1974.
14. Goodman, J. R., "Layered Wood Systems with Interlayer Slip", Ph.D. Dissertation, University of California, Berkeley, 1967.

15. Ko, M. F., "Layered Beam Systems with Interlayer Slip", M. S. Thesis, Colorado State University, Fort Collins, CO, March 1974.
16. Kuo, M. L., "Verification of a Mathematical Model for Layered T-Beams", M. S. Thesis, Colorado State University, Fort Collins, CO, March 1974.
17. Miller, I., and Freund, J. E., Probability and Statistics for Engineers, Prentice-Hall, Inc. Englewood Cliffs, New Jersey, 1965.
18. Span Tables for Joists and Rafters, recommended by National Forest Products Association, Sept. 1970.

APPENDIX A

The maximum deflection probability distributions derived by Monte Carlo analysis are approximations of actual distributions for the total population of floors. To assure that the approximation is accurate, the magnitudes of the error of the estimate (17) of the distribution means were computed for the simulation results.

Two assumptions were required for computation of the errors. The first assumption is that the deflection distribution was approximately normal. The second assumption is that the sample standard deviation is an acceptable approximation of the population standard deviation.

The magnitude of the error of the estimate for a probability of $1 - \alpha$ is

$$E < Z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

where $E = \bar{x} - \mu$ (error of the estimate)

s = sample standard deviation

$1 - \alpha$ = probability value

n = sample size

$Z_{\alpha/2}$ = area under the normal curve to the right of $\alpha/2$

For a probability of .9, the magnitude of the error of the estimate for each simulator case is listed in Table A.1. The errors of the estimate indicate that the Monte Carlo analysis provides good approximations of the actual distributions.

TABLE A.1

ERROR OF ESTIMATES OF MONTE CARLO APPROXIMATION

<u>SIMULATION</u>	<u>COEFFICIENT OF VARIATION</u>	<u>MEAN MAXIMUM DEFLECTION (in)</u>	<u>MAGNITUDE OF ERROR OF ESTIMATE</u>	
			<u>Magnitude</u>	<u>Percentage of Deflection Mean</u>
I	0.056	0.275	0.0017	0.6
I	0.204	0.296	0.0029	1.0
I	0.408	0.345	0.0066	1.9
II	0.056	0.335	0.0001	0.03
II	0.204	0.374	0.0040	1.1
II	0.408	0.454	0.0093	2.0
III	0.056	0.399	0.0014	0.4
III	0.204	0.436	0.0055	1.3
III	0.408	0.519	0.0130	2.5

